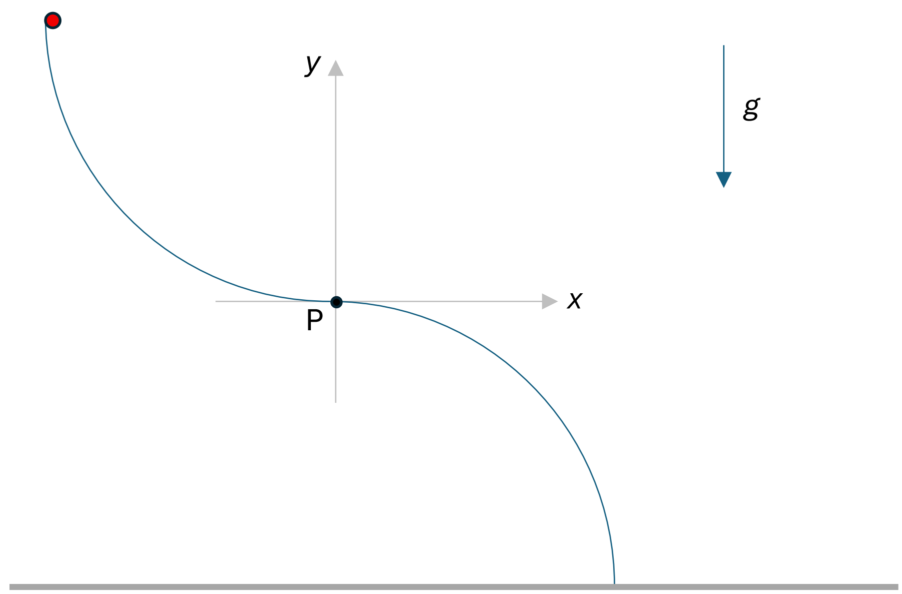
Teacher notes
Topic A

A parabolic motion problem

A path consists of two quarter circles of radius R as shown. A small ball is released from rest at the top. Air resistance and frictional forces are negligible.



(a) What is the speed of the ball when it reaches point P where the quarter circles join?
(b) What is the normal force on the ball just before P and right after P?
(c) Sketch a graph to show how the normal force varies with distance travelled along the top quarter circle.
(d) What happens to the ball after P?
(e) What angle does the velocity make with the horizontal at impact with the ground?

Answers

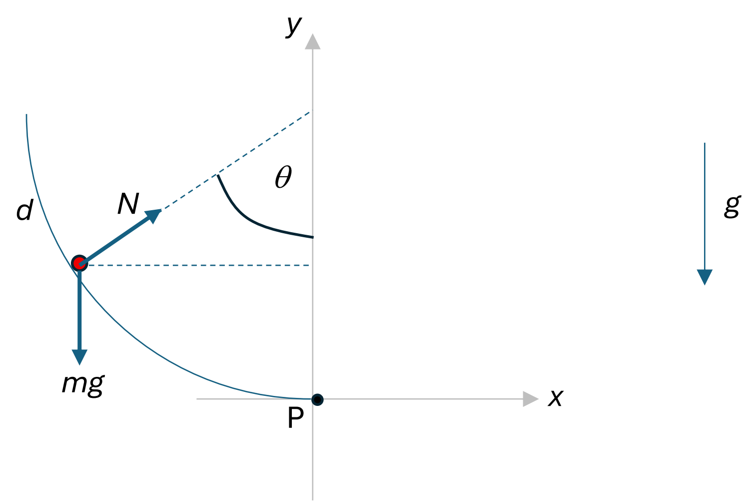
(a) $mgR = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gR}$

(b) Right before P, $N - mg = m\frac{v^2}{R} \Rightarrow N = mg + m\frac{v^2}{R} = mg + m\frac{2gR}{R} = 3mg$.

Right after P, $N = 0$, because the ball loses contact with the surface.

(c)

The diagram shows the ball when the line from the ball to the center of the upper quarter circle makes an angle θ to the vertical.



Then $N - mg\cos\theta = m\frac{v^2}{R} \Rightarrow N = mg\cos\theta + m\frac{v^2}{R}$.

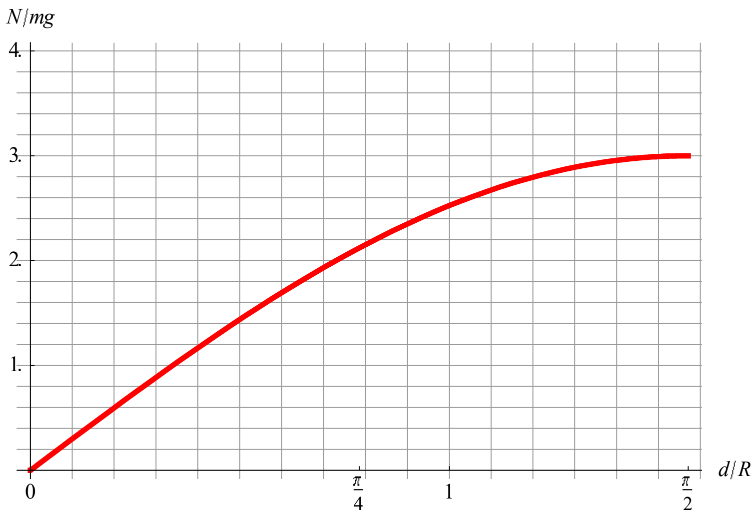
At this position, $mgR\cos\theta = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gR\cos\theta}$.

Hence $N = 3mg\cos\theta$.

From the diagram, $\frac{\pi}{2} - \theta = \frac{d}{R}$ where d is the distance travelled along the arc.

Finally, $N = 3mg\cos(\frac{\pi}{2} - \frac{d}{R})$.

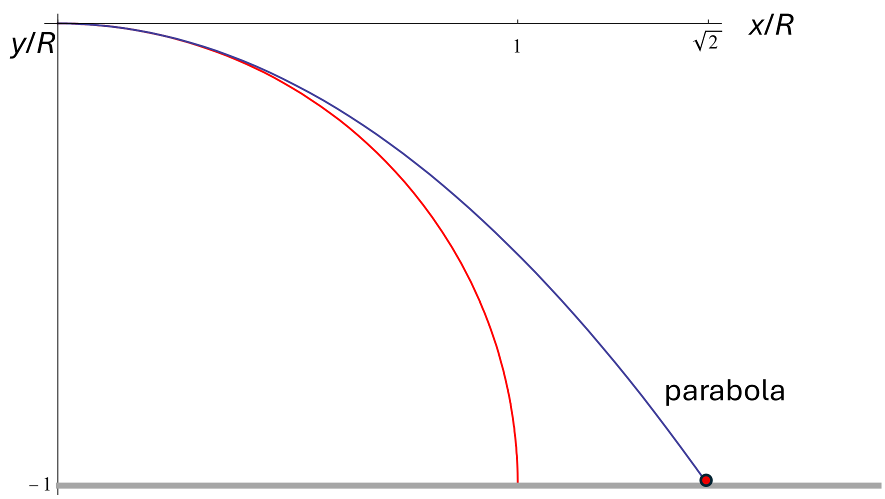
At the release point, $d = 0$ and $N = 0$. At the end of the quarter circle $d = \frac{\pi R}{2}$ and $N = 3mg$. The graph follows.



(d) At P the ball is launched horizontally with speed $v=\sqrt{2gR}$. The horizontal displacement is $x=vt$ and the vertical is $y=-\frac{1}{2}gt^2$. Combining the two gives

$$y=-\frac{1}{2}g(\frac{x}{v})^2=-\frac{1}{2}g\frac{x^2}{2gR} \text{ or } y=-\frac{1}{2}\frac{x^2}{R}$$, the equation of a parabola.

The equation of the quarter circle is $x^2+(y+R)^2=R^2$. The only common point of this quarter circle and the parabola is at x = 0 = y which means the ball loses contact with the surface after x = 0. The parabola is always above the quarter circle which means the ball lands to the right of the quarter circle on the ground. It lands when $y=-R$ and so from $y=-\frac{1}{2}\frac{x^2}{R}$ we get $-R=-\frac{1}{2}\frac{x^2}{R}\Rightarrow x=R\sqrt{2}$.



It is interesting that the distance at which the ball lands ($R\sqrt{2}$) does not depend on *g*.

(e) The horizontal and vertical velocity components at impact with the ground are both $v_x = v_y = \sqrt{2gR}$ and so the angle the velocity makes at impact is 45°.